Shore

Examination Number:

Set:

Year 12
Trial HSC Examination
August 2016

Mathematics

General Instructions

- Reading time − 5 minutes
- Working time – 3 hours
- Write using black pen
- Board-approved calculators may be used
- A BOSTES Reference Sheet is provided
- Answer Questions 1–10 on the Multiple Choice Answer Sheet provided
- In Questions 11−16, show relevant mathematical reasoning and/or calculations
- Start each of Questions 11−16 in a new writing booklet
- Write your examination number on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your examination number and "N/A" on the front cover

Total marks − 100

Section I Pages 2 – 5

10 marks

- Attempt questions 1–10
- Allow about 15 minutes for this section

Section II Pages 6 – 13

90 marks

- Attempt questions 11–16
- Allow about 2 hours and 45 minutes for this section

Note: Any time you have remaining should be spent revising your answers.

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Section I

10 marks
Attempt Questions 1−10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1−10.

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1 A computer costs \$1364.00 including 10% GST. What is the pre-GST cost of the computer?

(A) \$136.40

(B) \$1227.60

(C) \$1240.00

(D) \$1500.40

2 The quadratic equation $x^2+5x-2=0$ has roots α and β.
What is the value of $\alpha\beta-(\alpha+\beta)$?

(A) 3

(B) 7

(C) -3

(D) -7

3 What is the equation of the locus of a point that is always 5 units from the point $(2,-3)$?

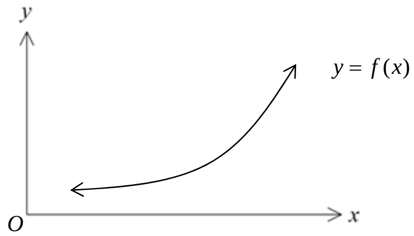
(A) $(x-2)^2+(y+3)^2=5$

(B) $(x+2)^2+(y-3)^2=25$

(C) $(x+2)^2+(y-3)^2=5$

(D) $(x-2)^2+(y+3)^2=25$

4 $y = f(x)$ is shown on the number plane.



Which of the following statements is true?

(A) $y = f(x)$ is decreasing and concave up.

(B) $y = f(x)$ is decreasing and concave down.

(C) $y = f(x)$ is increasing and concave up.

(D) $y = f(x)$ is increasing and concave down.

5 What are the solutions to $2\sin x = -\sqrt{3}$ for $0 \le x \le 2\pi$?

(A) $\frac{\pi}{3}$ and $\frac{2\pi}{3}$

(B) $\frac{4\pi}{3}$ and $\frac{5\pi}{3}$

(C) $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

(D) $\frac{4\pi}{3}$ and $\frac{2\pi}{3}$

6 If $a = \log_5 2$ and $b = \log_5 3$, what expression is equivalent to $\log_5 36$?

(A) $2(a+b)$

(B) $2ab$

(C) $a^2 + b^2$

(D) $(ab)^2$

7 The limiting sum of the geometric series $4 + 8x + 16x^2 + 32x^3 + \ldots$ is 140.

What is the value of x?

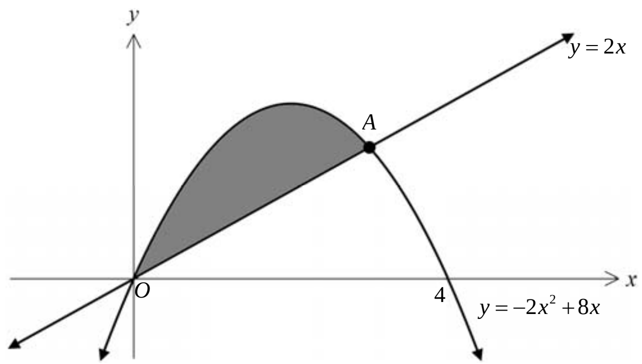
(A) $\frac{18}{35}$

(B) $\frac{1}{2}$

(C) $-\frac{1}{70}$

(D) $\frac{17}{35}$

8 The parabola $y = -2x^2 + 8x$ and the line $y = 2x$ intersect at the origin and at point A.



Which expression could be used to calculate the area enclosed by the parabola and the line?

(A) $\int_0^4 -2x^2 + 6x \, dx$

(B) $\int_0^4 -2x^2 + 8x \, dx - \int_0^3 2x \, dx$

(C) $\int_0^3 -2x^2 + 6x \, dx$

(D) $\int_0^3 -2x^2 + 8x \, dx - \int_0^4 2x \, dx$

9 The numbers 1 to 20 are written on cards and placed in a bag. One card is drawn at random.

What is the probability that the number on the card is even or a multiple of 3?

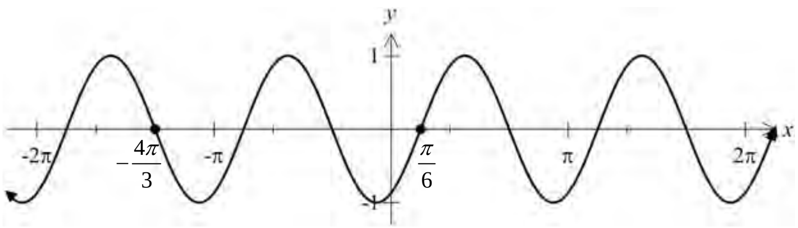
(A) $\frac{1}{2}$

(B) $\frac{4}{5}$

(C) $\frac{13}{20}$

(D) $\frac{7}{20}$

10 The graph shows the equation $y=\sin(Ax-B)$ over the domain $-2\pi \le x \le 2\pi$.



What are the values of A and B ?

(A) $A=2,\ B=\frac{\pi}{3}$

(B) $A=\frac{1}{2},\ B=\frac{\pi}{3}$

(C) $A=2,\ B=\frac{\pi}{6}$

(D) $A=\frac{1}{2},\ B=\frac{\pi}{6}$

Section II

90 marks
Attempt Questions 11−16
Allow about 2 hours and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11−16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPERATE writing booklet.

(a) Factorise x^3-8y^3. **1**

(b) Solve $|2x+1|=5$. **2**

(c) Express $\frac{1}{3-\sqrt{2}}$ with a rational denominator. **2**

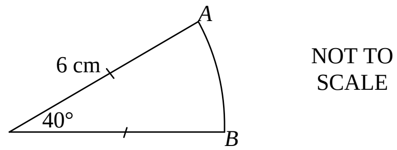
(d) Differentiate $3e^{x^2+1}$. **2**

(e) Differentiate $\frac{x^2}{5x+1}$. **2**

(f) Find a primitive of $\cos 2x$. **1**

(g) Find the exact value of $\int_0^1 \frac{x}{x^2+1}\, dx$. **3**

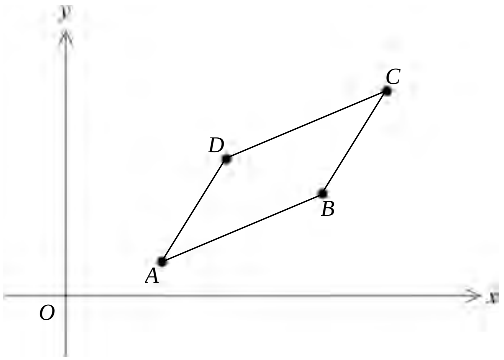
(h) The angle of a sector in a circle of radius 6 cm is 40°, as shown in the diagram below. **2**



Find the exact length of arc AB.

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) The points $A(3,1)$, $B(8,3)$, $C(10,6)$ and $D(5,4)$ are the vertices of a parallelogram.



NOT TO SCALE

(i) Find the length of interval *BC*. **1**

(ii) Show that the equation of *BC* is $3x-2y-18=0$. **2**

(iii) Find the perpendicular distance from *D* to *BC*. **2**

(iv) Hence, or otherwise, find the area of parallelogram *ABCD*. **1**

(b) 50 tickets are sold in a raffle. The raffle has two prizes, with the winning ticket not being replaced after each draw. Don has bought 4 tickets in the raffle.

(i) Find the probability Don wins only one prize. **2**

(ii) Find the probability Don does not win a prize. **1**

(iii) Find the probability Don wins at least one prize. **2**

(c) A parabola has equation $x^2-6x+8y+17=0$.

(i) Find the coordinates of the vertex of the parabola. **2**

(ii) Sketch the parabola, clearly showing the vertex, focus and directrix. **2**

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) The first three terms of an arithmetic series are -2, 0.5 and 3.

(i) Find the 24th term. **2**

(ii) Find the sum of the first 24 terms. **1**

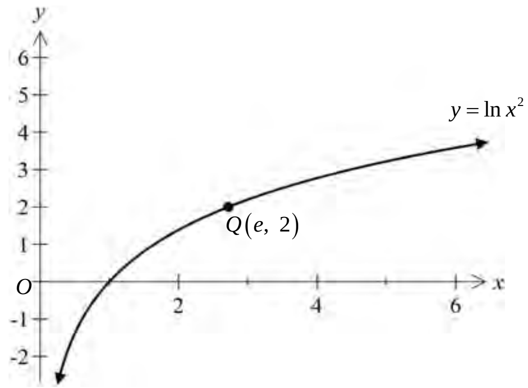
(b) Consider the curve $y=x^3+3x^2-9x-2$.

(i) Find any stationary points and determine their nature. **4**

(ii) Find the coordinates of any point(s) of inflexion. **2**

(iii) Sketch the curve labelling the stationary points, point of inflexion and *y*-intercept. **2**

(c) The point $Q(e,\ 2)$ lies on the curve $y=\ln x^2$.



(i) Use Simpson's rule with 5 function values to approximate $\int_1^5 \ln x^2\ dx$, correct to two decimal places. **2**

(ii) Find the equation of the tangent to the curve $y=\ln x^2$ at point *Q*. **2**

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) Find the values of k for which the equation $y = 3x^2 + kx - 4k$ is positive definite. **2**

(b) A particle moves in a straight line. It's velocity, V m/s, is given by the function $V = -3t^2 + 4t + 4$. Initially the particle is 3 metres to the right of the origin.

(i) Find when the particle is at rest. **2**

(ii) Find an expression for the displacement of the particle after t seconds. **2**

(iii) Find the displacement of the particle after 4 seconds. **1**

(iv) Find the total distance travelled by the particle in the first 4 seconds. **2**

(c) Henry borrows \$820 000 to purchase an apartment. The loan is to be repaid at a reducible interest rate of 4.8% p.a. The loan is repaid in monthly instalments of \$$M$.

(i) Show that the amount owing after 2 months, A_2, is given by **1**

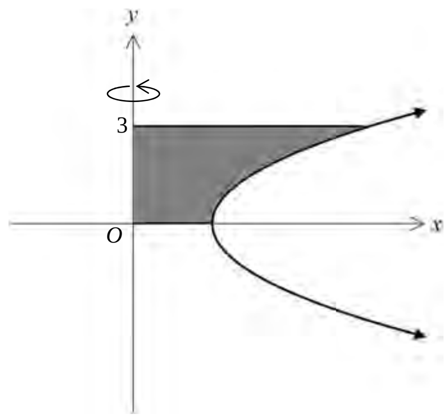
$$A_2 = 820\,000(1.004)^2 - M(1 + 1.004).$$

(ii) If the length of the loan is 25 years, show that the value of M, the monthly repayment, is \$4698.58. **2**

(iii) Instead, Henry makes monthly repayments of \$5100. **3**
After how many months will he have fully repaid the loan?

Question 15 (15 marks) Use a SEPARATE writing booklet.

(a) A parabola has equation $y^2 = x - 1$. **3**



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The area bounded by the curve $y^2 = x - 1$, the y-axis and the lines $y = 0$ and $y = 3$ is rotated about the y-axis to form a solid.

Find the volume of the solid.

(b) The value, \$$V$, of a car after t years is given by the equation $V = Ae^{-kt}$, where A and k are positive constants which depend on the make and model of the car.

(i) Kate buys a new hatchback for \$15 000. After 1 year her car is valued at \$13 000. **2**

Show that, for Kate's car, $k = 0.143$ correct to 3 significant figures.

(ii) At the same time Brian buys a new sports car. Its value is given by the equation $V = 20\,000e^{-0.2t}$. **3**

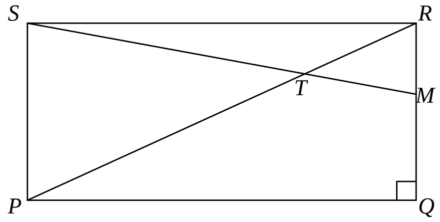
Find how long it is before Brian's and Kate's cars have the same value.

(iii) At what rate is the value of Brian's car decreasing after 3 years? **2**

Question 15 continues on the following page

Question 15 (continued)

(c) $PQRS$ is a rectangle and lines PR and SM intersect at T.
Point M divides RQ in the ratio 1 : 2.



NOT TO SCALE

Copy or trace the diagram into your writing booklet.

(i) Show that $\Delta MTR \, ||| \, \Delta STP$. **2**

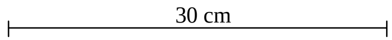
(ii) Given PR = 30 cm, find the length of RT. **3**

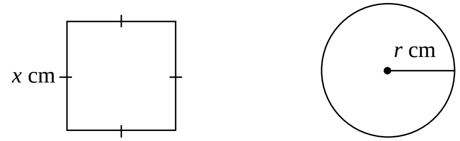
End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\displaystyle\int_{-\log_e 3}^{0} \frac{4}{e^{2x}} \, dx$. **3**

(b) A 30 cm length of wire is used to make two frames. The wire is to be cut into two parts. One part is bent into a square of side x cm and the remaining length is bent into a circle of radius r cm.





(i) The circumference of a circle, C, is found using the formula $C = 2\pi r$. **1**
Show that the expression for r in terms of x is $r = \dfrac{15 - 2x}{\pi}$.

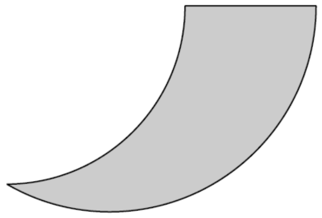
(ii) Show that the combined area, A, of the two shapes can be written as **2**
$A = \dfrac{(4+\pi)x^2 - 60x + 225}{\pi}$.

(iii) Find the value of x for which the combined area of the two frames will be minimised. Give your answer correct to 2 significant figures. **3**

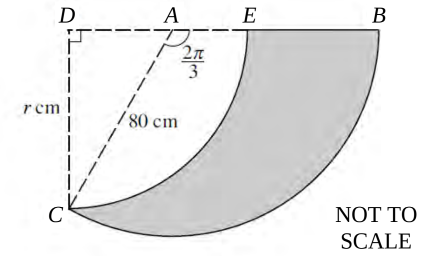
Question 16 continues on the following page

Question 16 (continued)

(c) The diagram below shows the cross-section of a rudder.



rudder



detail of construction

BC is an arc of a circle with centre A and radius 80 cm. $\angle CAB = \frac{2\pi}{3}$.

EC is an arc of a circle with centre D and radius r cm. $\angle CDE$ is a right angle.

(i) Show the area of sector ABC is $\frac{6400\pi}{3}$ cm^2. **1**

(ii) Show that $r = 40\sqrt{3}$. **2**

(iii) Hence, or otherwise, calculate the area of the cross-section of the rudder, correct to two decimal places. **3**

End of paper

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Year 12 Trial - 2 unit - 2016

Multiple Choice

1 C 2 A 3 D 4 C 5 B
6 A 7 D 8 C 9 C 10 A

1. $x \times 1.1 = 1364$
$x = 1240$
C

2. $x^2 + 5x - 2$
$\alpha\beta = -\frac{2}{1} = -2$ $\alpha + \beta = \frac{-5}{1} = -5$
$\alpha\beta - (\alpha + \beta) = -2 - (-5) = 3$
A

3. centre $(2, -3)$ radius 5
$(x-2)^2 + (y+3)^2 = 5^2$
$(x-2)^2 + (y+3)^2 = 25$
D

4. concave up, positive gradient
C

5. $2\sin x = -\sqrt{3}$
$\sin x = -\frac{\sqrt{3}}{2}$

$x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

$x = \pi + \frac{\pi}{3},\ 2\pi - \frac{\pi}{3}$
$= \frac{4\pi}{3},\ \frac{5\pi}{3}$
B

6. $a = \log_5 2$ $b = \log_5 3$
$\log_5 36 = \log_5 4 + \log_5 9$
$= \log_5 2^2 + \log_5 3^2$
$= 2\log_5 2 + 2\log_5 3$
$= 2(\log_5 2 + \log_5 3)$
$= 2(a+b)$
A

7. $S_\infty = \frac{a}{1-r}$ $r = \frac{8x}{4} = 2x$
$140 = \frac{4}{1-2x}$
$140 - 280x = 4$
$-280x = -136$
$x = \frac{17}{35}$
D

8. $A(3,6)$ $A = \int_0^3 -2x^2 + 8x - 2x$
$= \int_0^3 -2x^2 + 6x$
C

9. $P(\text{even or} \times 3) = P(\text{even}) + P(\times 3) - P(\text{both})$
$= \frac{10}{20} + \frac{6}{20} - \frac{3}{20}$
$= \frac{13}{20}$
C

10. $y = \sin\left[2\left(x - \frac{\pi}{6}\right)\right]$ $\therefore A = 2$
$= \sin\left(2x - \frac{\pi}{3}\right)$ $B = \frac{\pi}{3}$
A

Question 11

a) $x^3 - 8y^3 = x^3 - (2y)^3$
$= (x - 2y)(x^2 + 2xy + 4y^2)$

b) $|2x+1| = 5$
$2x + 1 = 5$ or $2x + 1 = -5$
$2x = 4$ $2x = -6$
$x = 2$ $x = -3$

c) $\frac{1}{3 - \sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{3+\sqrt{2}}{3^2 - (\sqrt{2})^2}$
$= \frac{3+\sqrt{2}}{9-2}$
$= \frac{3+\sqrt{2}}{7}$

d) $\frac{d}{dx} 3e^{x^2+1} = 3e^{x^2+1} \times 2x$
$= 6xe^{x^2+1}$

e) $\frac{d}{dx}\frac{x^2}{5x+1} = \frac{vu' - uv'}{v^2}$ $u = x^2$ $u' = 2x$ $v = 5x+1$ $v' = 5$
$= \frac{2x(5x+1) - 5x^2}{(5x+1)^2}$
$= \frac{10x^2 + 2x - 5x^2}{(5x+1)^2}$
$= \frac{5x^2 + 2x}{(5x+1)^2}$
$= \frac{x(5x+2)}{(5x+1)^2}$

f) $\int \cos 2x = \frac{\sin 2x}{2} + c$

g) $$\int_0^1 \frac{x}{x^2+1}\,dx = \left[\frac{1}{2}\ln|x^2+1|\right]_0^1$$

$$= \frac{1}{2}\left[\ln|1^2+1| - \ln|0^2+1|\right]$$

$$= \frac{1}{2}\left[\ln 2 - \ln 1\right]$$

$$= \frac{1}{2}\ln 2$$

$$= \ln\sqrt{2}$$

h) $40° = 40 \times \frac{\pi}{180}$ radians

$$= \frac{2\pi}{9}$$

Arc length $= r\theta$

$$= 6 \times \frac{2\pi}{9}$$

$$= \frac{4\pi}{3} \text{ cm}$$

QUESTION 12

a) i) $B(8,3)$, $C(10,6)$

$$BC = \sqrt{(10-8)^2 + (6-3)^2}$$

$$= \sqrt{4+9}$$

$$= \sqrt{13}$$

ii) $m_{BC} = \frac{6-3}{10-8}$

$$= \frac{3}{2}$$

$$\therefore\ y-6 = \frac{3}{2}(x-10)$$

$$2y-12 = 3(x-10)$$

$$2y-12 = 3x-30$$

$$0 = 3x-2y-18$$

iii) $d = \frac{|ax+by+c|}{\sqrt{a^2+b^2}}$ $\quad 0 = \overset{a}{3}x - \overset{b}{2}y \overset{c}{-18}$, $D(\overset{x}{5},\overset{y}{4})$

$$= \frac{|3\times5 + -2\times4 + -18|}{\sqrt{3^2+(-2)^2}}$$

$$= \frac{|15-8-18|}{\sqrt{13}}$$

$$= \frac{|-11|}{\sqrt{13}}$$

$$= \frac{11}{\sqrt{13}}$$

iv) $A = bh$

$$= \sqrt{13} \times \frac{11}{\sqrt{13}}$$

$$= 11\,u^2$$

b) i) $P_{(WL)} + P_{(LW)} = \frac{4}{50} \times \frac{46}{49} + \frac{46}{50} \times \frac{4}{49}$

$= \frac{184}{1225}$

ii) $P_{(LL)} = \frac{46}{50} \times \frac{45}{49}$

$= \frac{207}{245}$

iii) $P(\text{at least one}) = 1 - \frac{207}{245}$

$= \frac{38}{245}$

c) $x^2 - 6x + 8y + 17 = 0$

i) $x^2 - 6x = -8y - 17$

$x^2 - 6x + 9 = -8y - 17 + 9$

$(x-3)^2 = -8y - 8$

$(x-3)^2 = -8(y+1)$

$\therefore$ vertex $(3,-1)$

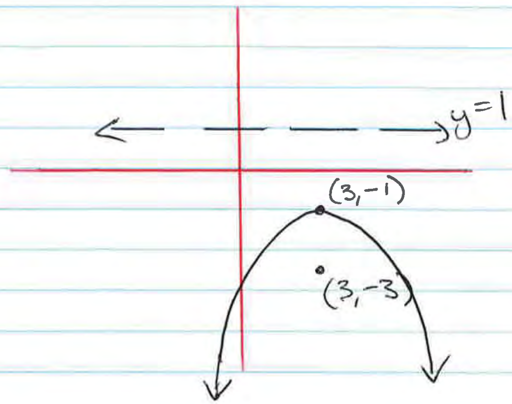
ii) vertex $(3,-1)$

$a = 2$

focus $(3,-3)$

directrix $y = 1$

sad face parabola



QUESTION 13

a) $-2, 0.5, 3$

i) $a = -2$, $d = 3 - 0.5$
$= 2.5$

$T_{24} = -2 + (24-1) \times 2.5$
$= 55.5$

ii) $S_n = \frac{n}{2}(a + l)$
$= \frac{24}{2}(-2 + 55.5)$

$= 642$

b) $y = x^3 + 3x^2 - 9x - 2$

i) $y' = 3x^2 + 6x - 9$

$y'' = 6x + 6$

stat. points when $y' = 0$

$0 = 3x^2 + 6x - 9$

$0 = x^2 + 2x - 3$

$0 = (x+3)(x-1)$

$\therefore x = -3, \quad x = 1$

when $x = -3$, $y = (-3)^3 + 3\times(-3)^2 - 9\times -3 - 2$
$= 25$

when $x = 1$, $y = (1)^3 + 3\times 1^2 - 9\times 1 - 2$
$= -7$

at $x = -3$ $y'' = 6\times -3 + 6$
$= -12$
< 0

$\therefore (-3, 25)$ is max t.p.

at $x = 1$ $y'' = 6\times 1 + 6$
$= 12$
> 0

$\therefore (1, -7)$ is min t.p.

ii) pt of inflexion when $y''=0$

$0 = 6x + 6$

$-6 = 6x$

$x = -1$

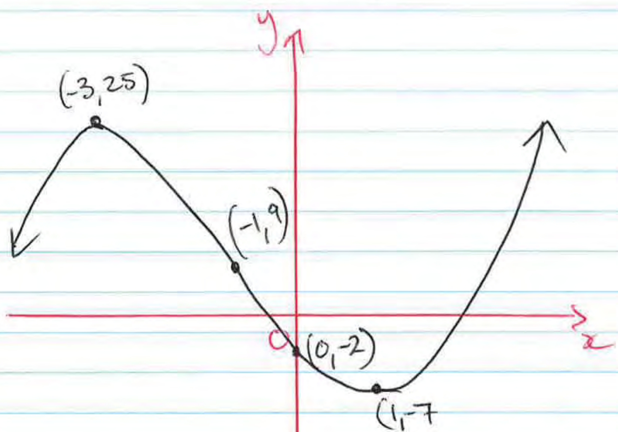
when $x=-1$, $y=9$

x	-2	-1	0
y''	-6	0	6

$\therefore$ change in concavity

$\therefore$ Point of inflexion at $(-1, 9)$

iii)



c) i)

x	1	2	3	4	5
y	$\ln 1$	$\ln 4$	$\ln 9$	$\ln 16$	$\ln 25$

$$\int_1^5 \ln x^2\,dx \doteqdot \frac{3-1}{6}\left(\ln 1 + 4\times\ln 4 + \ln 9\right) + \frac{5-3}{6}\left(\ln 9 + 4\times\ln 16 + \ln 25\right)$$

$\doteqdot 8.08295\ldots$

$\doteqdot 8.08$

ii) $y = \ln x^2$ $\qquad (e, 2)$

$y' = \frac{1}{x^2} \times 2x$

$= \frac{2}{x}$

at $x = e$, $y' = \frac{2}{e}$

$\therefore m = \frac{2}{e}$ $\quad$ Pt $(e, 2)$

$y - 2 = \frac{2}{e}(x - e)$

$ey - 2e = 2(x - e)$

$ey - 2e = 2x - 2e$

$0 = 2x - ey$

QUESTION 14

a) $y = 3x^2 + kx - 4k$

$\Delta < 0$

$k^2 - 4\times 3\times -4k < 0$

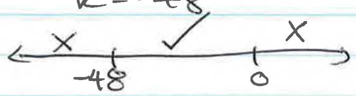
$k^2 + 48k < 0$

$k(k+48) < 0$

critical points inequality

$k(k+48) = 0$

$k = 0 \quad k = -48$



$\therefore -48 < k < 0$

b) i) $V = -3t^2 + 4t + 4$

at rest when $V = 0$

$0 = -3t^2 + 4t + 4$

$0 = -3t^2 + 6t - 2t + 4$

$0 = -3t(t-2) - 2(t-2)$

$0 = (t-2)(-3t-2)$

$\therefore t - 2 = 0 \qquad -3t = 2$

$t = 2 \qquad t = -\frac{2}{3}$

P -12
S 4
F 6, -2

$\therefore$ stationary when $t = 2$.

ii) displacement $= \int v$.

$\int -3t^2 + 4t + 4 \, dt = \frac{-3t^3}{3} + \frac{4t^2}{2} + 4t + c$

$= -t^3 + 2t^2 + 4t + c$

when $t = 0$, $x = 3$

$\therefore 3 = -0^3 + 2\times 0^2 + 4\times 0 + c$

$c = 3$

$\therefore x = -t^3 + 2t^2 + 4t + 3$

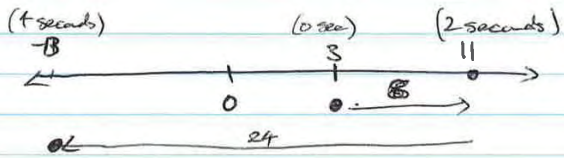
iii) $x = -4^3 + 2\times 4^2 + 4\times 4 + 3$

$= -13.$

iv) $V = 0$ at $t = 2$.

$\therefore$ at $t = 2 \quad x = -2^3 + 2\times 2^2 + 4\times 2 + 3$

$= 11$



$\therefore$ total distance $= (11-3) + (11 - -13)$

$= 8 + 24$

$= 32$

c) i) $A_1 = 820000 \times (1 + 0.004)^1 - M$ $\quad r = \frac{0.048}{12} = 0.004$

$= 820000 \times 1.004 - M$

$A_2 = A_1 \times 1.004 - M$

$= (820000 \times 1.004 - M) \times 1.004 - M$

$= 820000 \times 1.004^2 - M \times 1.004 - M$

$= 820000 \times 1.004^2 - M(1 + 1.004)$

ii) $A_n = 820000 \times 1.004^n - M(1 + 1.004 + \ldots + 1.004^{n-1})$

loan repaid after 25 years, $\therefore n = 300$

$\therefore A_{300} = 0$

$0 = 820000 \times 1.004^{300} - M(1 + 1.004 + \ldots + 1.004^{299})$

$0 = 820000 \times 1.004^{300} - M\left(\frac{1(1 - 1.004^{300})}{1 - 1.004}\right)$

$0 = 820000 \times 1.004^{300} - M \times 578.0448\ldots$

$M = \frac{820000 \times 1.004^{300}}{578.0448\ldots}$

$= 4698.5753\ldots$

$= \$4698.58$

iii) $A_n = 820000 \times 1.004^n - M(1 + 1.004 + \ldots + 1.004^{n-1})$

let $M = 5100$, $A_n = 0$

$0 = 820000 \times 1.004^n - 5100(1 + 1.004 + \ldots + 1.004^{n-1})$

$0 = 820000 \times 1.004^n - 5100\left(\frac{1(1.004^n - 1)}{1.004 - 1}\right)$

$0 = 820000 \times 1.004^n - 1275000(1.004^n - 1)$

$0 = 820000 \times 1.004^n - 1275000 \times 1.004^n + 1275000$

$-1275000 = 1.004^n(820000 - 1275000)$

$1.004^n = \frac{-1275000}{-455000}$

$\log_e 1.004^n = \log_e\left(\frac{255}{91}\right)$

$n \times \log_e 1.004 = \log_e\left(\frac{255}{91}\right)$

$n = \log_e\left(\frac{255}{91}\right) \div \log_e(1.004)$

$n = 258.1158\ldots$

$\therefore$ Repaid after 259 months.

QUESTION 15

a) $\int_0^3 [f(y)]^2\, dy$

$y^2 = x - 1$

$x = y^2 + 1$

$x^2 = (y^2+1)^2$

$x^2 = y^4 + 2y^2 + 1$

$\therefore V = \pi \times \int_0^3 y^4 + 2y^2 + 1 \; dy$

$= \pi \left[\frac{y^5}{5} + \frac{2y^3}{3} + 1y \right]_0^3$

$= \pi \times \left[\left(\frac{3^5}{5} + \frac{2\times 3^3}{3} + 3 \right) - \left(\frac{0}{5} + \frac{0}{3} + 0 \right) \right]$

$= \pi \times \frac{348}{5}$

$= \frac{348\pi}{5} \; u^3$

b) i) $V = Ae^{-kt}$

at $t=0$, $V=15000$ $\quad \therefore 15000 = Ae^0$

$A = 15000$

at $t=1$, $V=13000$

$\therefore 13000 = 15000e^{-k}$

$\frac{13000}{15000} = e^{-k}$

$\log_e\left(\frac{13}{15}\right) = -k$

$-0.1431.. = -k$

$k = 0.143$

iii) $V = 20000e^{-0.2t}$ $\qquad V = 15000e^{-0.143t}$

$20000e^{-0.2t} = 15000e^{-0.143t}$

$4e^{-0.2t} = 3e^{-0.143t}$

$\frac{4e^{-0.2t}}{3e^{-0.143t}} = 1$

$\frac{4e^{-0.2t-(-0.143t)}}{3} = 1$

$\frac{4e^{-0.057t}}{3} = 1$

$e^{-0.057t} = \frac{3}{4}$

$\log_e\left(\frac{3}{4}\right) = -0.057t$

$t = \frac{\log_e\left(\frac{3}{4}\right)}{-0.057}$

$t = 5.047..$

$\therefore$ after 5 years

iv) $V = 20000e^{-0.2t}$

$\frac{dV}{dt} = 20000e^{-0.2t} \times -0.2$

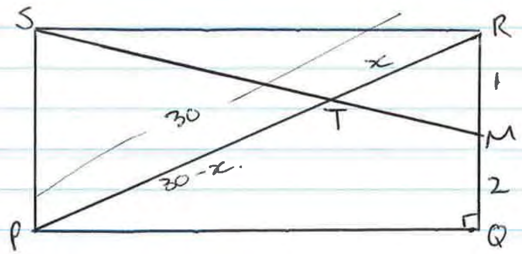
$= -4000e^{-0.2t}$

at $t=3$ $\quad \frac{dV}{dt} = -4000e^{-0.2\times 3}$

$= -4000e^{-0.6}$

$= -2195.25$

d)



i) In ΔMTR & ΔSTP

- $\angle MTR = \angle STP$ (vertically opposite $\angle$s are equal)
- $\angle TRM = \angle TPS$ (alternate $\angle$s are equal, $SP \parallel RQ$ opposite sides of a rectangle are equal)

$\therefore \Delta MTR \parallel\!\mid \Delta STP$ (equiangular)

ii) let $TR = x$, $\therefore TP = 30 - x$

$\therefore \frac{x}{30-x} = \frac{1}{3}$ $\qquad \frac{RT}{PT} = \frac{RM}{PS}$

$3x = 30 - x$

$4x = 30$

$x = 7.5$

$\therefore RT = 7.5\text{cm}$

QUESTION 16

a) $\int_{-\log_e 3}^{0} \frac{4}{e^{2x}}\,dx = \int_{-\log_e 3}^{0} 4e^{-2x}\,dx$

$= 4\left[\frac{e^{-2x}}{-2}\right]_{-\log_e 3}^{0}$

$= 4 \times \left[\frac{e^{-2\times 0}}{-2} - \frac{e^{-2\times -\log_e 3}}{-2}\right]$

$= 4 \times \left[\frac{1}{-2} + \frac{e^{2\log_e 3}}{2}\right]$

$= 4 \times \left[-\frac{1}{2} + \frac{9}{2}\right]$

$= 4 \times (4)$

$= 16$

b) i) $C = 2\pi r$

$C = 30 - 4x$

$\therefore 30 - 4x = 2\pi r$

$r = \frac{30-4x}{2\pi}$

$r = \frac{15-2x}{\pi}$

ii) $A = x^2 + \pi r^2$

$= x^2 + \pi \times \left(\frac{15-2x}{\pi}\right)^2$

$= x^2 + \pi \times \left(\frac{15^2 - \cancel{2}\cancel{80}60x + 4x^2}{\pi^2}\right)$

$= x^2 + \frac{225 - 60x + 4x^2}{\pi}$

$= \frac{\pi x^2 + 225 - 60x + 4x^2}{\pi}$

$= \frac{(4+\pi)x^2 - 60x + 225}{\pi}$

iii) $A = \frac{(4+\pi)}{\pi}x^2 - \frac{60}{\pi}x + 225$

$\frac{dA}{dx} = \frac{2(4+\pi)}{\pi}x - \frac{60}{\pi}$

$\frac{d^2A}{dx^2} = \frac{2(4+\pi)}{\pi}$, $\therefore > 0$, $\therefore$ always min t.p.

let $\frac{dA}{dx} = 0$ $\quad 0 = \frac{2(4+\pi)}{\pi}x - \frac{60}{\pi}$

$\frac{60}{\pi} = \frac{2(4+\pi)}{\pi}x$

$60 = 2(4+\pi)x$

$x = \frac{60}{2(4+\pi)}$

$x = \frac{30}{4+\pi}$ $\quad \frac{d^2A}{dx^2} > 0$

$\therefore x = 4.2$ will give minimum area

c) i) $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 80^2 \times \frac{2\pi}{3}$

$= \frac{6400\pi}{3}$

ii) $\angle CAD = \pi - \frac{2\pi}{3}$

$= \frac{\pi}{3}$ (straight $\angle$ is $180°$)

$\therefore \sin\frac{\pi}{3} = \frac{r}{80}$

$r = 80\sin\frac{\pi}{3}$

$r = 80 \times \frac{\sqrt{3}}{2}$

$= 40\sqrt{3}$

$\boxed{\text{OR}}$ $\frac{2\pi}{3}^c = \frac{2\pi}{3} \times \frac{180}{\pi}$ degrees

$= 120°$

$\therefore \angle CAD = 180 - 120$

$= 60°$ (straight $\angle$ is $180°$)

$\sin 60 = \frac{r}{80}$

$r = 80\sin 60$

$r = 40\sqrt{3}$

iii) Area of ΔADC

$AD = \sqrt{80^2 - (40\sqrt{3})^2}$

$= \sqrt{1600}$

$= 40$

$\therefore$ Area $\Delta ADC = \frac{1}{2} \times 40 \times 40\sqrt{3}$

$= 800\sqrt{3}$.

$\boxed{\text{OR}}$ $\therefore \angle ACD = 180 - 90 - 60$

$= 30°$

$\therefore$ Area $\Delta_{ADC} = \frac{1}{2} \times 80 \times 40\sqrt{3} \times \sin 30$

$= 1600\sqrt{3} \times \frac{1}{2}$

$= 800\sqrt{3}$

$\therefore$ Total Area = Area Sector ABC + Area Δ ADC − Area Quadrant DEC

$= \frac{6400\pi}{3} + 800\sqrt{3} - \frac{1}{4} \times \pi \times (40\sqrt{3})^2$

$= \frac{6400\pi}{3} + 800\sqrt{3} - 1200\pi$

$= 4317.7937\ldots$

$= 4317.79 \text{ cm}^2$